Massachusetts Institute of Technology - Physics Department

Physics - 8.01 **Final** Fall 1999

SOLUTIONS

Problem 1 16 points

4 pts **a)** $v_y = v_0 \sin\theta - gt = 0 \rightarrow \boldsymbol{t = \frac{v_0 \sin\theta}{g}}$

4 pts **b)** $y = v_0 + (v_0 \sin\theta)t - \frac{1}{2}gt^2$

$y = 0 + (v_0 \sin\theta)\left(\frac{v_0 \sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta}{g}\right)^2$

$\boldsymbol{y = \frac{1}{2}\frac{v_0^2 \sin^2\theta}{g}}$

4 pts **c)** $\boldsymbol{v_0}$

4 pts **d)** $x = v_0 + (v_0 \cos\theta)t \qquad t = 2\frac{v_0 \sin\theta}{g}$

$\boldsymbol{x = \frac{2v_0^2 \sin\theta\cos\theta}{g}}$

<u>Problem 2</u> 15 points

5 pts **a)** $mgl = \frac{1}{2}mv_A^2 \rightarrow \boldsymbol{v_A = \sqrt{2gl}}$

5 pts **b)** Net force up must be $ma_{\text{cen}} = m\frac{v_A^2}{l} = 2mg$

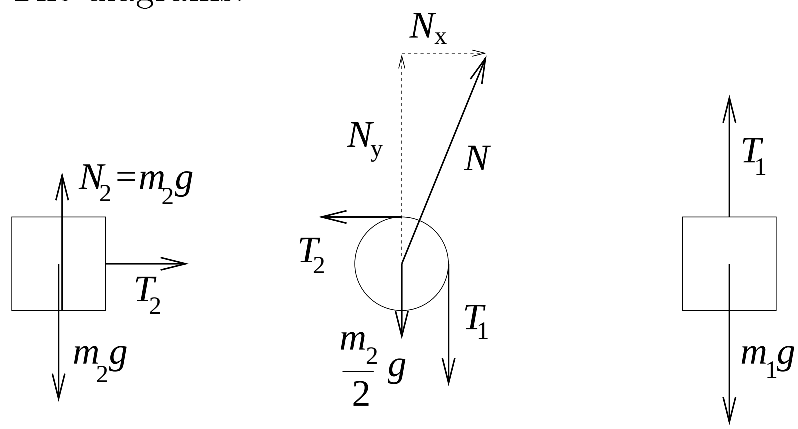
$T - mg = 2mg \rightarrow \boldsymbol{T = 3mg}$

5 pts **c)** gravity: $\boldsymbol{mgl}$

tension: **0**

Problem 3 24 points

6 pts **a)** The diagrams:



The contact force N on the pulley must have components $N_{\mathrm{x}} = T_2$ and $N_{\mathrm{y}} = \frac{m_2}{2} g + T_1$ for the pulley to stay in place.

6 pts **b)** $T_2 = m_2 a = \boldsymbol{m_2 \left(\frac{g}{2}\right)}$

6 pts **c)** $R(T_1 - T_2) = I\alpha \qquad I = \frac{1}{2}\frac{m_2}{2} R^2 = \frac{m_2 R^2}{4} \qquad \alpha = \frac{a}{R}$

$R(T_1 - m_2 \frac{g}{2}) = I \frac{a}{R} = \frac{Ig}{2R} = \frac{m_2 g R}{8}$

$\Rightarrow T_1 = \frac{m_2 g}{8} + \frac{m_2 g}{2} = \boldsymbol{\frac{5}{8} m_2 g}$

6 pts **d)** $m_1 g - T_1 = m_1 a = m_1 \frac{g}{2}$

$m_1 \frac{g}{2} = T_1 = \frac{5}{8} m_2 g$

$\Rightarrow \boldsymbol{m_1 = \frac{5}{4} m_2}$

Problem 4 20 points

4 pts **a)** $\tau_p = |\vec{r_p} \times F| = \boldsymbol{-bMg\sin\theta}$

4 pts **b)** $I_p = I_c + Mb^2 = \boldsymbol{\frac{1}{2}MR^2 + Mb^2}$

4 pts **c)** $\sum \tau_p = I_p\alpha$

$-bMg\sin\theta = (\frac{1}{2}MR^2 + Mb^2)\ddot{\theta}$

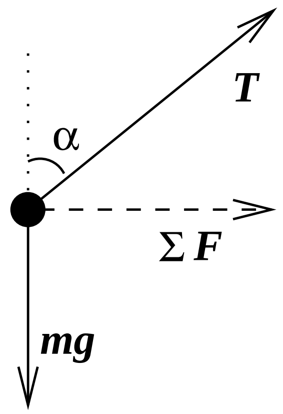
$\Rightarrow \boldsymbol{\ddot{\theta} + \frac{bg}{\frac{1}{2}R^2+b^2}\sin\theta = 0}$

4 pts **d)** Under the small angle approximation, $\sin\theta \approx \theta$, and the equation of motion is given by $\ddot{\theta} + \frac{bg}{\frac{1}{2}R^2+b^2}\theta = 0$ which is simple harmonic with an angular frequency given by $\omega = \sqrt{\frac{bg}{\frac{1}{2}R^2+b^2}}$. The period is given by $T = \frac{2\pi}{\omega} = \boldsymbol{2\pi\sqrt{\frac{\frac{1}{2}R^2+b^2}{bg}}}$.

4 pts **e)** There must be a force at P or the CM would accelerate straight downwards.

<u>Problem 5</u> 25 points

4 pts **a)** The diagram:



4 pts **b)** $\boldsymbol{\omega = \frac{2\pi}{\tau}}$

$v = \omega R \rightarrow \boldsymbol{\vec{v} = \frac{2\pi R}{\tau}\hat{z}}$

4 pts **c)** $a = \frac{v^2}{R} \rightarrow \boldsymbol{\vec{a} = \frac{4\pi^2 R}{\tau^2}\hat{x}}$

4 pts **d)** $\vec{F} = m\vec{a} \rightarrow \boldsymbol{\vec{F} = \frac{m4\pi^2 R}{\tau^2}\hat{x}}$

9 pts **e)** $T\sin\alpha = \frac{m4\pi^2 R}{\tau^2}$

$T\cos\alpha = mg$

$\rightarrow \boldsymbol{\tan\alpha = \frac{4\pi^2 R}{g\tau^2}}$

Problem 6 25 points

3 pts **a)** $\boldsymbol{\vec{p} = m_1 v_1} \hat{\mathrm{x}}$

4 pts **b)** $\boldsymbol{\vec{p}' = m_1 v_1} \hat{\mathrm{x}}$

4 pts **c)** $\boldsymbol{\frac{1}{2} m_1 v_1^2}$

6 pts **d)** $m_1 v_1' \sin\theta_1 = m_2 v_2' \sin\theta_2 \rightarrow \boldsymbol{\frac{v_2'}{v_1'} = \frac{m_1 \sin\theta_1}{m_2 \sin\theta_2}}$

8 pts **e)** $m_1 v_1' \cos\theta_1 + m_2 v_2' \cos\theta_2 = m_1 v_1$

$m_1 v_1' \cos\theta_1 + \left(m_1 v_1' \frac{\sin\theta_1}{\sin\theta_2}\right) \cos\theta_2 = m_1 v_1$

$v_1' \left(\cos\theta_1 + \frac{\sin\theta_1}{\sin\theta_2} \cos\theta_2\right) = v_1$

$\rightarrow \boldsymbol{v_1' = v_1 \frac{\sin\theta_2}{\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2}}$

Problem 7 25 points

4 pts **a)** $\frac{1}{2}mv^2 = \frac{mMG}{R} \rightarrow \boldsymbol{v = \sqrt{\frac{2MG}{R}}}$

4 pts **b)** $mv_0 R \sin 30° = mV(15R) \rightarrow \boldsymbol{\frac{v_0}{V} = \frac{15}{\sin 30°} = 30}$

4 pts **c)** $\boldsymbol{\frac{1}{2}mv_0^2 - \frac{mMG}{R}}$

5 pts **d)** **same as in c.** or $\boldsymbol{\frac{1}{2}mV^2 - \frac{mMG}{15R} = \frac{1}{2}m\frac{v_0^2}{900} - \frac{mMG}{15R}}$

8 pts **e)** $\frac{1}{2}mv_0^2 - \frac{mMG}{R} = \frac{1}{2}m\frac{v_0^2}{900} - \frac{mMG}{15R}$

$\boldsymbol{\frac{1}{2}v_0^2 - \frac{MG}{R} = \frac{1}{2}\frac{v_0^2}{900} - \frac{MG}{15R}}$

Problem 8 18 points

4 pts **a)** $\boldsymbol{M = \rho V}$

6 pts **b)** $W' = W - Mg + F_{\text{buoy}} \qquad F_{\text{buoy}} = Mg$

$W' = W - Mg + Mg \rightarrow \boldsymbol{W' = W}$

8 pts **c)** $mg + T = F_{\text{buoy}} = Mg \rightarrow \boldsymbol{T = Mg - mg}$

$\boldsymbol{W' = W - Mg + F_{\text{buoy}} - T = W - T}$

Problem 9 15 points

$$\frac{dP}{dy} = -\rho g \qquad \rho = \frac{\text{mass}}{\text{volume}} = \frac{Nm}{V} = \frac{P}{kT}m$$

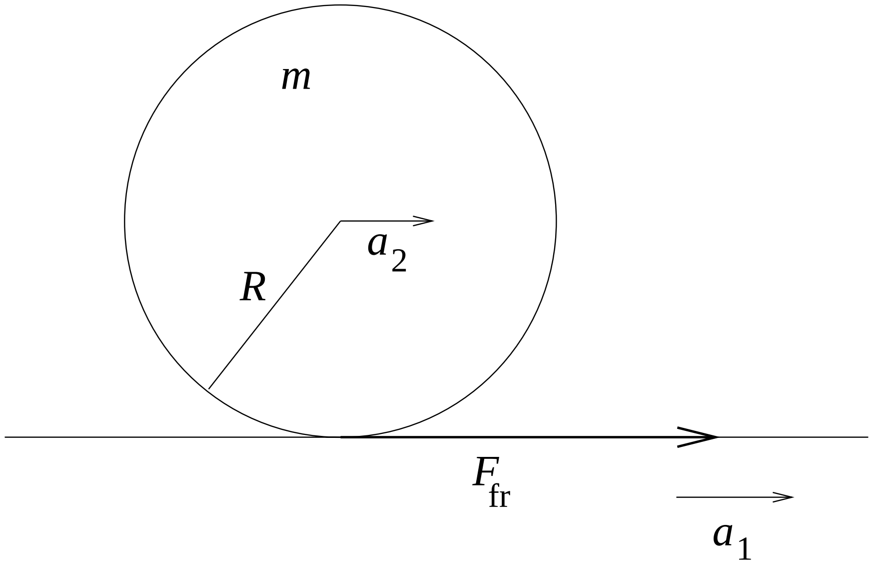
$$\frac{dP}{dy} = -\frac{Pm}{kT}g \rightarrow \frac{dP}{P} = -\frac{mg}{kT}dy$$

$$\int_{P_0}^{P} \frac{dP}{P} = \int_0^h -\left(\frac{mg}{kT}\right) dy$$

$$\log \frac{P}{P_0} = -\frac{mgh}{kT}$$

$$\boldsymbol{P = P_0 \mathrm{e}^{-\frac{mgh}{kT}}}$$

Problem 10 17 points



$$\sum F = ma \qquad \sum \tau = I\alpha \qquad \text{no slipping :}$$

$$F_{\text{fr}} = ma_2 \qquad RF_{\text{fr}} = \frac{2}{5}mR^2\alpha \qquad a_1 - a_2 = \alpha R$$

$$F_{\text{fr}} = \frac{2}{5}mR\alpha$$

$$ma_2 = \frac{2}{5}m(a_1 - a_2)$$

$$\frac{7}{5}a_2 = \frac{2}{5}a_1$$

$$\boldsymbol{a_2 = \frac{2}{7}a_1}$$