Title: Absolute Maximum proper time to an initial event, its curvature and matter

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Abstract

Einstein equation of Gravity has on one side the momentum energy density tensor and on the other, Einstein tensor which is derived from Ricci curvature tensor.
A better theory of gravity will have both sides geometric.
Another goal should be to describe time as perpendicular unlike in Kerr solution, in order to show it is an emergent dimension rather than an ordinary dimension like the other 3. To do that we need to use a measure of time that is independent of the choice of coordinates.
To summarize, this paper has two purposes as follows: first, to show a model of matter as merely due to possible but not inevitable geometric conflict, second, to show the way to see time as an emergent dimension. The second purpose was not achieved though the paper does show new ideas.

Keywords: Foliation, Field Curvature, General Relativity, Accelerated Cosmic expansion, Quantum Gravity

1. Introduction: Square Field Curvature in positive definite metric spaces

A) Ideas

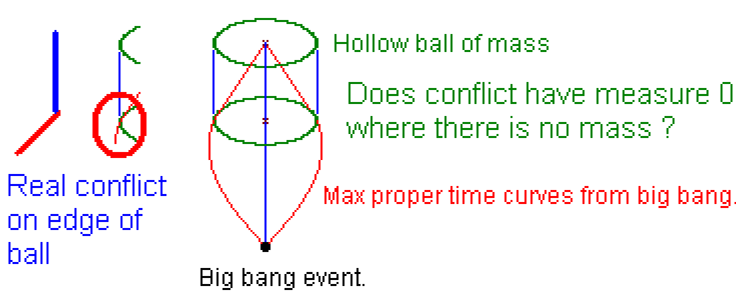
Deconstructing space time to 3+1 dimension requires time orthogonality.

If spacetime is homotopic to a single starting event, say "big bang" then maximal proper time curves can be drawn between that event and any other event and therefore attach a time value to any event in space time. This time then becomes a scalar field. Because clock tick is different under local space translations, the scalar field has a significant gradient by space.

The gradient need not be parallel to any geodesic curve

The direction in space time of the maximum proper time forms a geodesic curve but not necessarily the gradient of the field is parallel to a geodesic curve. A good example is the center and edge of a hollowed ball of mass. Due to General Relativity, the clock ticks in the gravitational field of the ball are slower than far from the ball. As a result, max proper time geodesic curves from say "big bang" event, must come from outside the ball into the ball. The time at the center of the ball is also a geodesic curve but it is in the time direction in Schwarzschild coordinates. The vector field of the lines is therefore discontinuous and we have a non zero [1] Euler number.

(Fig. 1) The line of the max proper time field from "big bang" is discontinuous in the middle of a hollowed ball of mass. In microscopic level, the gradient apparently bends on the edge where matter exists.



Contrary to the absolute maximal proper time from "big bang", most geodesic curves usually measure only local maximum proper time. The curves of the global/absolute maximum proper time have tangents that are eigenvectors of the metric tensor, corresponding to the maximum eigenvalue. Therefore, in geodesic coordinates such that the time is parallel to the maximum proper time, the mixed terms of the metric tensor vanish. Locally, the separation between space and time works also in metrics such as the Kerr metrics and time appears perpendicular to 3D space manifolds along the maximum proper time curves. Separation of space and time is important, however, this paper has a higher priority motivation, to get an equation that depends only on geometry. To show that time is an emergent dimension is secondary in this paper.

B) Questions – second, emergent time unsolved issue,

The question is: Can inverted logic work ? By minimizing an action operator on three dimensional manifolds, can a degree of freedom yield multiple solutions for the metric tensor, such that:

1) The action can serve as a homotopy [2] parameter.

2) The action will be invariant under Lorentz - like rotations in the resulting four dimensional manifold

C) Numerical results

A C++ simulation was written and tried on a 40x40x40 grid and yielded an expanding three dimensional domain.

We would like to describe the curvature of the gradient of the absolute maximum proper time from "big bang" scalar field and show its possible relation to Ricci curvature and to Einstein's tensor. The idea is that the gradient of the scalar time field of absolute maximum proper time from "Big Bang", forms curves that have non vanishing curvature.

Intuitive discussion about the second power of curvature of a conserving vector field

So let us begin. We will now define what the Square Field Curvature FC of a vector field V in R^n, with positive definite Euclidean geometry, is.The same formalism is easily extended to Riemannian geometry.

We would like the field V to reduce or increase its differential in directions that are perpendicular to the direction of the field. This requirement is also comprehensible when the metric tensor of a manifold with coordinates in R^n has only positive eignevalues in local orthogonal coordinates and we shall see that the operator that describes Field Curvature has quite the same formalism in Riemannian manifolds. We will start with an intuitive description of the operator and later give a proof it is the square curvature of the vector field. Given two infinitesimally close points in R^n, $q1$ and $q2 = q1 + hV$ for some infinitesimal h, we would like that $V(q2) - V(q1)$ will be as parallel as possible to the field $V(q1)$.

By Pythagoras that can be written as the following problem locally minimize

$$\left((\frac{V(q2)-V(q1)}{h}) \bullet (\frac{V(q2)-V(q1)}{h}) - ((\frac{V(q2)-V(q1)}{h}) \bullet \frac{V(q1)}{|V(q1)|})^2 \right) h^2 \qquad \textbf{(1)}$$

When $\bullet$ is the inner product in R^n.

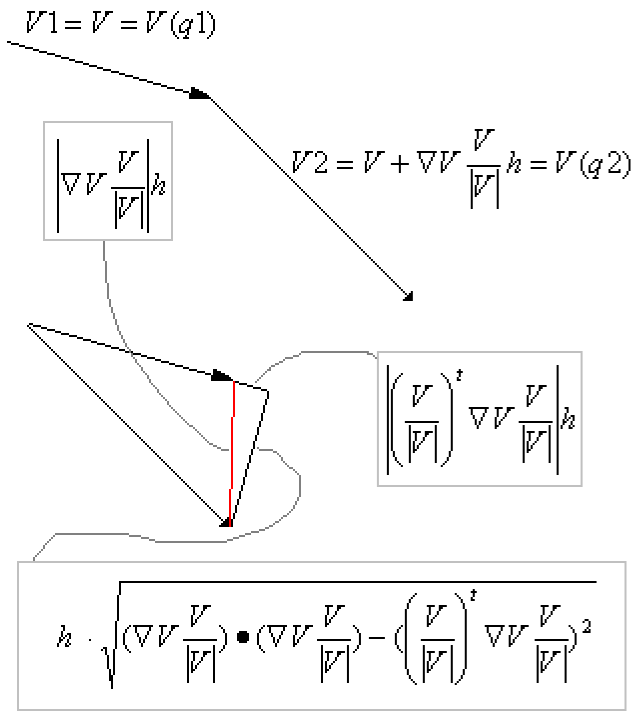
Here $(V(q2)-V(q1)) \bullet \frac{V(q1)}{|V(q1)|}$ represents the projection of the derivative matrix of the vector field $V(q)$ multiplied by the field direction in space.

In other words, since h^2 is arbitrarily small, our objective is to minimize,

$$(\nabla V \bullet \frac{V}{|V|}) \bullet (\nabla V \bullet \frac{V}{|V|}) - (\left(\frac{V}{|V|}\right)^t \bullet \nabla V \bullet \frac{V}{|V|})^2 = \frac{(\nabla V \bullet V) \bullet (\nabla V \bullet V)}{V \bullet V} - (\frac{V^t \bullet \nabla V \bullet V}{V \bullet V})^2 \qquad \textbf{(2)}$$

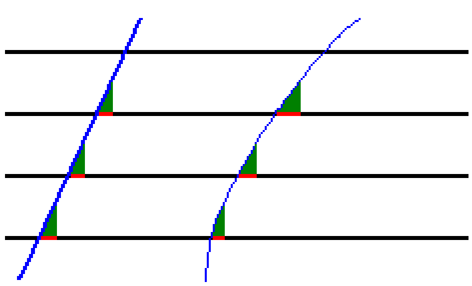
Here ∇V means the matrix $a_{ij} = \dfrac{\partial V_i}{\partial X^j}$.

(Fig. 2) - Field Curvature and its Euclidean geometric meaning – how much the field changes in direction perpendicular to itself.



The following next figure shows us two curves one on the left for which BE is zero and one on the right for which BE is positive:

(Fig. 3) - Parallel deviation on the right.



2. Tensor formalism of the Square Curvature

As will be discussed, in tensor formalism, derivatives are replaced by covariant derivatives and are denoted by semi colon and derivatives by comma. Upper and lower indices represent the covariant and contra-variant properties and upper and lower indices sum according to Einstein convention so (2) can be written as a tensor density with local coordinates in R^n. Regarding the metrics square root of the determinant of the metric tensor $\sqrt{-g}$, so following are tensor densities [3], that yield tensor equations [4],

$$SquareCurvature \equiv \frac{1}{4}\left(\frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{m}}(\mathrm{P}^{s}\mathrm{P}_{s}),_{\mathrm{k}}\, g^{mk}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^2} - \frac{((\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{m}}\,\mathrm{P}^{m})^2}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^3} \right)\sqrt{-g} \text{ or} \qquad \textbf{(3)}$$

that can be written as $\frac{1}{4}\left(V_{\mathrm{m}}V^{m} - (\frac{1}{\mathrm{t}})^2\right)\sqrt{g}$ for $V_m = \frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{m}}}{\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}}}$ and $\frac{1}{t} = \frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{m}}\,\mathrm{P}^{m}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^{3/2}}$.

Don't' confuse t with maximum proper time. We choose a simpler expression for our absolute maximum proper time from "big bang" τ ,

$$\boxed{P = \tau}. \qquad \textbf{(3A)}$$

Explanation to the seemingly impossible non geodesic gradient

Question: Can $\mathrm{P}_{\lambda} = \frac{dP}{dx^{\lambda}}$ be **non-** tangent to a geodesic curve ?

1) If there is more than one geodesic curve that connects the "Big Bang" event to say event 'e', then obviously P_{λ} need not be geodesic in a small neighborhood of 'e'.
2) P as a value is not the length of a local coordinate !!!

Development of (3) can be from the following:

$$\boxed{\begin{aligned} L &= \frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{m}}(\mathrm{P}^{s}\mathrm{P}_{s}),_{\mathrm{k}}\, g^{mk}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^2} - \frac{((\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{m}}\,\mathrm{P}^{m})^2}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^3} = U^{j}U_{j} \\ U_m &= \frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{m}}}{\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}}} - \frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mu}\,\mathrm{P}^{\mu}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^2}P_m \end{aligned}} \qquad \textbf{(3.1)}$$

Obviously $U_m P^m = 0$.

The vector U_m describes the direction and intensity of the curvature of the field P_λ which is a change perpendicular to P^m.

From minimum action of

$$\begin{array}{l} Z = \mathrm{P}_\mu \mathrm{P}^\mu \text{ and } U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2} \text{ and } \mathrm{L} = \mathrm{U}^\kappa \mathrm{U}_\mathrm{k} \\ R = Ricci \text{ curvature.} \\ Action = Min \int_\Omega (R + L).\sqrt{-g}\, d\Omega \end{array}$$

$$\begin{array}{l} L = U_i U^i \text{ and } Z = \mathrm{P}^\mathrm{k} \mathrm{P}_\mathrm{k} \\ (-2(\frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_\mathrm{m} P^m}{Z^3} P^k);_k P^\mu P^\nu - 2\frac{(Z_s P^s)^2}{Z^3}) \frac{P^\mu P^\nu}{Z} + \\ \frac{1}{2}\frac{(P^\lambda Z_\lambda)^2}{Z^3} g^{\mu\nu} - \frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P^\mu P^\nu}{P^i P_i} - \\ (-2(\frac{Z^m}{Z^2});_m P^\mu P^\nu - 2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P^\mu P^\nu}{Z} + \frac{1}{2}\frac{Z_\mathrm{k} Z^k}{Z^2} g^{\mu\nu} - \frac{Z^\mu Z^\nu}{Z^2}) = \\ R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} \end{array} \quad \textbf{(3.2)}$$

and assuming $(\frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_\mathrm{m} P^m}{Z^3} P^k);_k - (\frac{Z^m}{Z^2});_m = 0$ we have $-L = -R$

such that $R_{\mu\nu}$ is the Ricci curvature tensor [5]. [6]. In other words, curvature of the gradient of absolute maximum proper time from "Big Bang" (as a possible result of more than one geodesic curve) is equivalent to Ricci curvature.

If that can be true then we can have an equation that is based solely on geometry.

In any case we have a nice action (without spinors [7] and other fancy mathematical technology) of the form:

$V^\lambda V_\lambda - \frac{1}{t^2}$ such that V_λ is a vector field and $\frac{1}{t}$ is a scalar field. If the definition is in 3 dimensions, it hints at 4 dimensional Lorentzian metric geometry.

U_μ is units of $\frac{1}{Length}$. Just as a qualitative argument we can justify the minus sign in

$V^\lambda V_\lambda - \frac{1}{t^2}$ because $\begin{array}{l} r^2 - x^2 - y^2 - z^2 \geq 0,\ r>0, x>0, y>0, z>0 \\ \Leftrightarrow \frac{1}{r^2} \leq \frac{1}{x^2+y^2+z^2} \leq \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} \Rightarrow \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} - \frac{1}{r^2} \geq 0 \end{array}$.

A rigid proof that SquareCurvature is the square (to the second power of) field curvature

We restrict the proof to the Euclidean case. The square curvature is defined as

$$Curv^2 \equiv \frac{d}{dt}\left(\frac{V_\lambda}{\sqrt{V^k V_k}}\right)\frac{d}{dt}\left(\frac{V_\mu}{\sqrt{V^k V_k}}\right)g^{\lambda\mu} \qquad \textbf{(3.4)}$$

such that $g^{\lambda\mu}$ is a diagonal unit matrix. For convenience we will write $Norm \equiv \sqrt{V^k V_k}$

and $\dot{V}_\lambda \equiv \frac{d}{dt}V_\lambda$. For some parameter t. Let W_λ denote:

$$W_\lambda = \frac{d}{dt}\left(\frac{V_\lambda}{\sqrt{V^k V_k}}\right) = \frac{\dot{V}_\lambda}{Norm} - \frac{V_\lambda}{Norm^3}V_k \dot{V}_\nu g^{k\nu} \qquad \textbf{(3.5)}$$

Obviously

$$W_\lambda V_k g^{\lambda k} = \frac{\dot{V}_\lambda V_k g^{\lambda k}}{Norm} - \frac{V_\lambda V_s g^{\lambda s}}{Norm^3}V_k \dot{V}_\nu g^{k\nu} = \frac{\dot{V}_\lambda V_k g^{\lambda k}}{Norm} - \frac{V_k \dot{V}_\nu g^{k\nu}}{Norm} = 0 \qquad \textbf{(3.6)}$$

Thus

$$Curv^2 = W_\lambda W^\lambda = \frac{\dot{V}_\lambda \dot{V}_\nu g^{\lambda\nu}}{Norm^2} - \frac{V_\lambda \dot{V}_s g^{\lambda s}}{Norm^4}V_k \dot{V}_\nu g^{k\nu} = \frac{\dot{V}_\lambda \dot{V}^\lambda}{Norm^2} - \left(\frac{V_\lambda \dot{V}^\lambda}{Norm^2}\right)^2 \qquad \textbf{(3.7)}$$

Since $\frac{V_\lambda}{Norm}$ is the derivative of the normalized curve or normalized “speed”,

$\frac{d}{dt}V_\lambda = \left(\frac{d}{dx^r}V_\lambda\right)\frac{dx^r}{dt} = \frac{d}{dx^r}V_\lambda \frac{V^r}{Norm} \equiv V_{\lambda},_r \frac{V^r}{Norm}$ such that x^r denotes the local coordinates.

If V_λ is a conserving field then $V_{\lambda},_{r} = V_{r},_{\lambda}$ and thus $V_{\lambda},_{r} V^{r} = \frac{1}{2} Norm^2,_{\lambda}$

$$Curv^2 = \frac{\dot{V}_\lambda \dot{V}^\lambda}{Norm^2} - (\frac{V_\lambda \dot{V}^\lambda}{Norm^2})^2 = \frac{1}{4}\frac{Norm^2,_{\lambda}\, Norm^2,_{k}\, g^{\lambda k}}{Norm^4} - \frac{1}{4}(\frac{Norm^2,_{s} V_r g^{sr}}{Norm^3})^2 \quad \textbf{(3.8)}$$

Writing the last term in Riemannian geometry is the same field curvature operator that we chose.

Geodesic coordinates where the time coordinate is parallel to the global/absolute maximum proper time

We see that 3 dimensions hint at 4 dimensional action. This is done by looking at the action (3) in three dimensions and observing the following way to write it,

$$\begin{pmatrix} \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_{\mathrm{m}} \mathrm{P}^m}{(\mathrm{P}^{\mathrm{i}} \mathrm{P}_{\mathrm{i}})^{\frac{3}{2}}} & \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_0}{\mathrm{P}^{\mathrm{i}} \mathrm{P}_{\mathrm{i}}} & \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_1}{\mathrm{P}^{\mathrm{i}} \mathrm{P}_{\mathrm{i}}} & \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_2}{\mathrm{P}^{\mathrm{i}} \mathrm{P}_{\mathrm{i}}} \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & g^{00} & g^{01} & g^{02} \\ 0 & g^{10} & g^{11} & g^{12} \\ 0 & g^{20} & g^{21} & g^{22} \end{pmatrix} \begin{pmatrix} \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_{\mathrm{m}} \mathrm{P}^m}{(\mathrm{P}^{\mathrm{i}} \mathrm{P}_{\mathrm{i}})^{\frac{3}{2}}} \\ \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_0}{\mathrm{P}^{\mathrm{i}} \mathrm{P}_{\mathrm{i}}} \\ \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_1}{\mathrm{P}^{\mathrm{i}} \mathrm{P}_{\mathrm{i}}} \\ \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_2}{\mathrm{P}^{\mathrm{i}} \mathrm{P}_{\mathrm{i}}} \end{pmatrix}$$

$$g^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & g^{00} & g^{01} & g^{02} \\ 0 & g^{10} & g^{11} & g^{12} \\ 0 & g^{20} & g^{21} & g^{22} \end{pmatrix} \text{and } \mathrm{q}^{\mathrm{ij}} = \begin{pmatrix} g^{00} & g^{01} & g^{02} \\ g^{10} & g^{11} & g^{12} \\ g^{20} & g^{21} & g^{22} \end{pmatrix}$$

(3.9)

Where $g^{\mu\nu}$ is the metric tensor in 4 dimensions and q^{ij} is in 3. q^{ij} implicitly refers to a local submersion [8] where time is locally held constant.

Can we do the opposite, look at 4 dimensions and reduce the problem to 3 without violating the principle of covariance ?

First, our maximum proper time curves are intrinsic and do not depend on the coordinates.

We can therefore agree that the maximum proper time curves are different than ordinary geodesic curves on which only local maxima of proper time can be measured.

We choose to describe (3) on our space-time in our special coordinates. Since the direction in space time of the maximum proper time is an eigenvector of the metric tensor with the biggest eigenvalue, our metric tensor is of the form presented in (3.9) for which the mixed space time terms are zero. Also,

$\mathrm{P} = \tau \Rightarrow \mathrm{P}_0 = 1 \Rightarrow P_\mu P^\mu = -1 + P_\lambda P^\lambda, \lambda = 1,2,3$ and

$\mathrm{P}_{0},_{1} = \mathrm{P}_{0},_{2} = \mathrm{P}_{0},_{3} = \mathrm{P}_{1},_{0} = \mathrm{P}_{2},_{0} = \mathrm{P}_{3},_{0} = 0$

We can assume as possible $\mathrm{P}_1 \neq 0, \mathrm{P}_2 \neq 0, \mathrm{P}_3 \neq 0$ especially if multiple maximum proper time curves to the same event 'e' exist.

So instead of (3) we reduce the action to become three dimensional,

$$\text{Tweaked } SquareCurvature \equiv \frac{1}{4}\left(\frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_{\mathrm{m}} (\mathrm{P}^s \mathrm{P}_s),_{\mathrm{k}}\, g^{mk}}{(\text{-}1+\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^2} - \frac{((\mathrm{P}^\lambda \mathrm{P}_\lambda),_{\mathrm{m}} \mathrm{P}^m)^2}{(\text{-}1+\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^3}\right)\sqrt{g} \text{ or}$$

$$\text{Tweaked } BE \equiv \frac{1}{4}\left(\frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_{\mathrm{m}} (\mathrm{P}^s \mathrm{P}_s),_{\mathrm{k}}\, g^{mk}}{(\text{-}1+\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})} - \left(\frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_{\mathrm{m}} \mathrm{P}^m}{(\text{-}1+\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})}\right)^2\right)\sqrt{g} = \qquad \textbf{(4)}$$

$$(\text{-}1+\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}}) \cdot SquareCurvature$$

This means that on our three dimensional sub-manifolds ("Leaves of a foliation"), there is a corresponding action operator that is free of derivative dependence on time. Solving the Euler Lagrange equations for the Tweaked Square Curvature and receiving a plurality of solutions is indeed a promising direction of research !!!

Unsynchronizability

Since P is not constant on the 3 dimensional sub-manifolds perpendicular to the global/absolute maximal proper time curves, these manifolds are not synchronizable and are therefore not the ideal inflating S(3) i.e. Friedmann-Robertson-Walker.

History of the paper's concept of time

The idea of an absolute time, such as maximum proper time from a common event i.e. "big bang" is not new [9], [10] and it appears in Hebrew writing such as the Book of Principles by Rabbi Josef Albo 1380-1444. Rabbi Josef Albo wrote about time that can be measured by devices and another aspect of time which he termed immeasurable. The maximum proper time can't be measured by devices on Earth because due to General Relativity, clock ticks are slowed down by the gravitational field.

Euler Lagrange Equations of the SquareCurvature action

We will not solve the entire system

$$\begin{array}{l} \mathrm{Z} = \mathrm{P}_{\mu}\mathrm{P}^{\mu} \text{ and } U_{\lambda} = \dfrac{Z_{\lambda}}{Z} - \dfrac{Z_k P^k P_{\lambda}}{Z^2} \text{ and } \mathrm{L} = \mathrm{U}^{\kappa}\mathrm{U}_{\mathrm{k}} \\ R = Ricci \text{ curvature.} \\ \delta\int_{\Omega}\left(R + \mathrm{U}^{\kappa}\mathrm{U}_{\mathrm{k}}\right)\sqrt{g}\,d\Omega = 0 \end{array} \qquad \textbf{(4.1)}$$

But rather focus on $\delta\int_{\Omega}\left(\mathrm{U}^{\kappa}\mathrm{U}_{\mathrm{k}}\right)\sqrt{g}\,d\Omega$

$$L=\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}\ \text{s.t.}\ Z=\mathrm{P}_{\mu}\mathrm{P}^{\mu}$$

$$\frac{\partial L}{\partial g_{\mu\nu}}-\frac{d}{dx^{m}}\frac{\partial L}{\partial g_{\mu\nu},_{m}}=$$

$$\left(\begin{array}{l}
2(\frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{s}}P^{s}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^{3}})\left(P^{\mu};_{m}P^{\nu}P^{m}+P^{\mu}P^{\nu};_{m}P^{m}-\Gamma_{i\ \ m}^{\ \mu}P^{i}P^{\nu}P^{m}-\Gamma_{i\ \ m}^{\ \nu}P^{\mu}P^{i}P^{m}\right)+\\
-3(\frac{((\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{s}}P^{s})^{2}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^{4}})P^{\mu}P^{\nu}+\\
\frac{1}{2}(\frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{m}}P^{m}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^{3/2}})^{2}g^{\mu\nu}+\\
-2(\frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{s}}P^{s}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^{3}})((P^{\mu}P^{\nu}P^{m});_{m}-P^{i}P^{\nu}P^{m}\Gamma_{i\ \ m}^{\ \mu}-P^{\mu}P^{i}P^{m}\Gamma_{i\ \ m}^{\ \nu})+\\
-2\frac{((\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{s}}P^{s}),_{m}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^{3}}(P^{\mu}P^{\nu}P^{m})+\\
+6(\frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{s}}P^{s}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^{4}})(P^{\mu}P^{\nu}P^{m})(P^{r}P_{r}),_{m}
\end{array}\right)\sqrt{g}=$$

$$\left(\begin{array}{l}
(-2(\frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{m}}P^{m}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^{3}}P^{k});_{k}P^{\mu}P^{\nu}-2\frac{(Z_{\mathrm{s}}P^{s})^{2}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^{3}})\frac{P^{\mu}P^{\nu}}{P^{i}P_{i}}+\\
\frac{1}{2}\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}g^{\mu\nu}-\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}\frac{P^{\mu}P^{\nu}}{P^{i}P_{i}}
\end{array}\right)\sqrt{g} \tag{4.2}$$

$$L = \frac{Z^{\lambda} Z_{\lambda}}{Z^2} \text{ s.t. } Z = P_{\mu} P^{\mu}$$

$$\frac{\partial L}{\partial g_{\mu\nu}} - \frac{d}{dx^m}\frac{\partial L}{\partial g_{\mu\nu},_m} =$$

$$\left(\begin{array}{l} 2\frac{P^{\mu};_m P^{\nu} Z^m + P^{\mu} P^{\nu};_m Z^m - \Gamma_i{}^{\mu}{}_m P^i P^{\nu} Z^m - \Gamma_i{}^{\nu}{}_m P^{\mu} P^i Z^m}{(P^i P_i)^2} + \\ \frac{(P^{\lambda} P_{\lambda}),_m (P^s P_s),_k (-\frac{1}{2}(g^{m\mu} g^{\nu k} + g^{m\nu} g^{\mu k}))}{(P^i P_i)^2} + \\ -2\frac{(P^{\lambda} P_{\lambda}),_m (P^s P_s),_k g^{mk}}{(P^i P_i)^3} P^{\mu} P^{\nu} + \\ \frac{1}{2}\frac{(P^{\lambda} P_{\lambda}),_m (P^s P_s),_k g^{mk}}{(P^i P_i)^2} g^{\mu\nu} \\ \\ -2\frac{((P^{\mu} P^{\nu} Z^m);_m - P^i P^{\nu} Z^m \Gamma_i{}^{\mu}{}_m - P^{\mu} P^i Z^m \Gamma_i{}^{\nu}{}_m)}{(P^i P_i)^2} + \\ \\ +4\frac{(P^{\mu} P^{\nu} Z^m)(P^r P_r),_m}{(P^i P_i)^3} \end{array} \right) \sqrt{g} =$$

$$((-2(\frac{Z^m}{Z^2});_m P^{\mu} P^{\nu} - 2\frac{Z^{\lambda} Z_{\lambda}}{Z^2}\frac{P^{\mu} P^{\nu}}{P^i P_i} + \frac{1}{2}\frac{Z_k Z^k}{Z^2} g^{\mu\nu} - \frac{Z^{\mu} Z^{\nu}}{Z^2})\sqrt{g} \tag{4.3}$$

$$Z = P_{\mu} P^{\mu} \text{ and } U_{\lambda} = \frac{Z_{\lambda}}{Z} - \frac{Z_k P^k P_{\lambda}}{Z^2} \text{ and } L = U^{\kappa} U_k$$

$$\frac{\partial L}{\partial g_{\mu\nu}} - \frac{d}{dx^m}\frac{\partial L}{\partial g_{\mu\nu},_m} =$$

$$\left(\begin{array}{l} (-2(\frac{(P^{\lambda} P_{\lambda}),_m P^m}{(P^i P_i)^3} P^k);_k P^{\mu} P^{\nu} - 2\frac{(Z_s P^s)^2}{(P^i P_i)^3})\frac{P^{\mu} P^{\nu}}{P^i P_i} + \\ \frac{1}{2}\frac{(P^{\lambda} Z_{\lambda})^2}{Z^3} g^{\mu\nu} - \frac{(P^{\lambda} Z_{\lambda})^2}{Z^3}\frac{P^{\mu} P^{\nu}}{P^i P_i} - \\ ((-2(\frac{Z^m}{Z^2});_m P^{\mu} P^{\nu} - 2\frac{Z^{\lambda} Z_{\lambda}}{Z^2}\frac{P^{\mu} P^{\nu}}{P^i P_i} + \frac{1}{2}\frac{Z_k Z^k}{Z^2} g^{\mu\nu} - \frac{Z^{\mu} Z^{\nu}}{Z^2}) \end{array} \right) \cdot \sqrt{g} \tag{4.4}$$

$$L = \frac{(Z^s P_s)^2}{Z^3} \text{ s.t. } \mathrm{Z} = \mathrm{P}^\lambda \mathrm{P}_\lambda \text{ and } \mathrm{Z_m} = (\mathrm{P}^\lambda \mathrm{P}_\lambda)_{,m}$$

$$\frac{\partial L}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial L}{\partial P_{\mu,\nu}} =$$

$$+4(Z_s P^s)\frac{(P^\mu{}_{;\nu} P^\nu - \Gamma_i{}^\mu{}_k P^i P^k)}{Z^3}\sqrt{g} +$$

$$-4(Z_s P^s)\frac{(\mathrm{P}^\mu P^\nu)_{;\nu} - \Gamma_i{}^\mu{}_\nu P^i P^\nu}{Z^3}\sqrt{g} +$$

$$-4(Z_s P^s)_{,\nu}\frac{\mathrm{P}^\mu P^\nu}{Z^3}\sqrt{g} +$$

$$+12(Z_s P^s)\frac{(\mathrm{P}^\mu P^\nu)}{Z^4} Z_\nu \sqrt{g} +$$

$$+2\frac{Z_m P^m Z^\mu}{Z^3}\sqrt{g} +$$

$$-3\frac{(Z_m P^m)^2}{Z^4} P^\mu \sqrt{g} =$$

$$(-4(\frac{(Z_s P^s)P^\nu}{Z^3})_{;\nu} P^\mu + 2\frac{Z_m P^m Z^\mu}{Z^3} - 3\frac{(Z_m P^m)^2}{Z^4} P^\mu)\sqrt{g} \tag{4.5}$$

$$L=\frac{Z^s Z_s}{Z^2}\ \text{s.t.}\ \mathrm{Z}=\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}\ \text{and}\ \mathrm{Z}_{\mathrm{m}}=(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{m}$$

$$\frac{\partial L}{\partial P_\mu}-\frac{d}{dx^\nu}\frac{\partial L}{\partial P_\mu,_\nu}=$$

$$4\frac{(P^\mu;_\nu Z^\nu-\Gamma_i{}^\mu{}_k P^i Z^k)}{Z^2}\sqrt{g}+$$

$$-4\frac{Z_{\mathrm{m}}Z^m}{Z^3}\mathrm{P}^\mu\sqrt{g}$$

$$-4\frac{((\mathrm{P}^\mu Z^\nu);_\nu-\Gamma_i{}^\mu{}_\nu P^i Z^\nu-\Gamma_i{}^\mu{}_\nu P^\nu Z^i)}{Z^2}\sqrt{g}+$$

$$8\frac{(\mathrm{P}^\mu Z^\nu)}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^3}(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}}),_\nu\sqrt{g}=$$

$$(-4(\frac{Z^\nu}{Z^2});_\nu-4\frac{Z_{\mathrm{m}}Z^m}{Z^3})\mathrm{P}^\mu\sqrt{g} \qquad \textbf{(4.6)}$$

Finally we get the following zero divergence:

$\frac{d}{dx^\mu}(\frac{\partial L}{\partial P_\mu}-\frac{d}{dx^\nu}\frac{\partial L}{\partial P_\mu,_\nu})(U_k U^k\sqrt{g})=W^\mu;_\mu\sqrt{g}=0$ where

W^μ is obtained from the subtraction of (4.5) from (4.6).

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