Adam S. Bolton
bolton@mit.edu

March 6, 2002

MIT 8.02 Spring 2002
Exam #1 Solutions

Problem 1

(a) No bulbs glowing: no closed circuit anywhere and hence no current anywhere.

(b) A and B glow with equal brightness, as they are connected in series to the battery and thus the same current passes through each. C is still off.

(c)A, B, and C all glow. A is brightest, for all current flows through it. B and C glow with equal but lesser brightness, as the current through A is split equally between B and C.

(d) Bulb A in case (c) is brightest of all: effective resistance of the bulb combination is decreased from that of part (b) by the addition of bulb C in parallel with bulb B. By Ohm's law, more current is then drawn from the battery in case (c) as compared to case (b), leading to a brighter bulb A.

Bulbs B and C in case (c) are faintest of all. Let V be the battery voltage and R be the resistance of each bulb. The effective resistance of the circuit as a whole is $2R$ in case (b) and $1.5R$ in case (c). Thus the current through A is $V/2R$ in case (b) and $V/1.5R = 2V/3R$ in case (c). Therefore in case (b) the current through B is also $V/2R$, but in case (c) the current through B (and C) is *half* of $2V/3R$, which is $V/3R$. This latter current is less than $V/2R$, and so B and C in case (c) are fainter than A and B in case (b).

(e-b) B glowing, C off.
(e-c) B and C glowing with equal brightness.
(e-d) All on-bulb brightnesses are equal, for all bulbs have the full battery voltage across themselves, and therefore the same current goes through each.

Problem 2

(a) $Q = CV$, and C & V are the same for both capacitors. So each has a charge $+CV$ on its upper plate and $-CV$ on its lower plate.

(b) For both capacitors, electric field has magnitude $E = V/d$ and is directed downwards (from + charge to −).

(c) Right capacitor: C & V unchanged, so $+CV$ on upper plate and $-CV$ on lower plate still. Left capacitor: V unchanged, but $C \to \kappa C = 3C$, giving charges $+3CV$ on upper plate and $-3CV$ on lower.

(d) Same answer as part (b), for V and d remain unchanged.

Problem 3

(a) Since there is no current inside the conductor, the E-field is zero everywhere inside. Therefore Gauss's law dictates that there cannot be any net charge inside the conductor, so *none* of the charge will be found in the region $r_1 < r < r_2$. Now suppose some of the net positive charge $+q$ remained on the *inner* surface of the pipe. We could enclose this charge with a Gaussian surface lying entirely within the conducting material (ignoring end effects), and since $\mathbf{E} = 0$ everywhere within a conductor, Gauss's law would tell us that our Gaussian surface contained no net charge. This would only be possible if there were some *negative* charge within the cavity to balance the positive charge on the inner pipe wall. But this cavity is *empty.* So we may conclude that none of the charge will stay on the inner wall. Since we have found that it cannot go anywhere else, *all* charge $+q$ must go to the outer surface. Symmetry of the system dictates that the charge will be distributed evenly over this surface, with surface charge density $\sigma = +q/2\pi r_2 L$. (See the solution to homework problem 2.1 for discussion of a related situation.)

(b) Symmetry requires that $\mathbf{E}$ be in the radial direction and depend only upon r. (By "radial" here we mean perpendicularly away from the pipe axis, not radial in the sense of spherical coordinates. Unfortunately there is no good terminology to distinguish between "cylindrical-radial" and "spherical-radial".) Application of Gauss's law to a cylindrical Gaussian surface of length l and radius r coaxial with the pipe leads to $(2\pi rl)E = Q_{\text{encl}}/\epsilon_0$. **(i)** For $r < r_1$, $Q_{\text{encl}} = 0$ and hence $\mathbf{E} = 0$. **(ii)** We already know that $\mathbf{E} = 0$ inside the conductor $(r_1 < r < r_2)$. **(iii)** For $r > r_2$, $Q_{\text{encl}} = +ql/L$, and thus $\mathbf{E} = +q/(2\pi\epsilon_0 rL)$ radially outward (i.e. cylindrical-radially).

(c) $\Delta V = 0$: potential difference is the line integral of the electric field, and the electric field is zero everywhere between the axis and the outer surface.

(d) Nothing changes from part (b): there was no E-field in the pipe cavity to begin with, so there is nothing to induce a polarization charge in the dielectric.

END